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Evaluation of Petrophysical Relations by Using Cluster Analysis

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Abstract

The paper describes the methodological aspects of building one-dimensional (1D) models of the physical and mechanical properties of rocks in the near-wellbore space through building statistical correlations of petrophysical and physical and mechanical properties based on the integrated field geological and geophysical information, including laboratory core studies, mud logging, and well logging data.

Key Words: geomechanical modeling, 1D geomechanical model, mechanical properties model, petrophysical correlations, multivariate regression, Principal Component Analysis, Young's modulus, Poisson's ratio, tensile strength, clustering, coring

Introduction

An increasingly relevant area in the oil and gas industry today is building and introducing digital geological and geomechanical models of fields based on spatial distribution (1D models along a wellbore or 3D sector models) of rock physical and mechanical properties and stress condition (Kashnikov, Yu.A. et al., 2019). Such models allow to address a wide range of tasks, such as calculating wellbore stability while drilling (determining the drilling window and the optimal reservoir entry angle, as well as stable state period); designing and implementing well completions (including cementing); stability of near-wellbore area during well operations (calculation of critical drawdown, solids and cumulative sand production at supercritical drawdowns); high-quality designing of well workovers (e.g., hydraulic fracturing); defining the orientation and magnitude of the maximum horizontal stress (building 3D models); taking into account the effect of changes in the rock stress during field operation on the rock properties and production parameters (building 4D models; Zhang, J., 2019).

Such models allow addressing a wide range of tasks (Toropetsky, K.V., Kayurov, N.K. et al., 2016):

1. Geomechanical properties study: defining the orientation and magnitude of the maximum horizontal stress (building 1D and 3D models); taking into account the effect of changes in the rock stress during filed operation on the flow properties of rocks and well production parameters (building 4D models).

2. Well construction: determining a safe range of drilling mud weights; drafting cementing programs; selecting the optimal reservoir entry angle; mapping risks and preventive measures to prevent or minimize the consequences.
3. Well completions: selecting downhole equipment and building integrated designs of completion assemblies; placing packers and ports for multi-stage hydraulic fracturing.
4. General reservoir engineering tasks: near-wellbore zone stability – estimating critical drawdowns, solids amount, and cumulative production of rock particles at various drawdowns.

Today, the greatest accuracy of the physical and mechanical properties' measurements (Young's modulus, Poisson's ratio, tensile strength, see below) can only be obtained by a lab core analysis. However, such tests are expensive and the amount of high-quality core material is limited. Moreover, there are rocks (rock salts, mudstones) that, for various reasons, are difficult to recover or to be analyzed at high quality (salt creep in static experiments, fracturing, etc.).

A normal practice to calculate the profile of physical and mechanical properties in wells (1D MMS) is to build multivariate regression models using a combination of laboratory core experiments, logging data interpretations, mud logging while drilling, seismic, well workover, etc. Lots of papers have been published on the calculation of physical and mechanical properties through correlations with logging/mud logging data (Toropetsky, K.V., Ulyanov, V.N. et al, 2016).

The purpose of this paper is to classify existing experience and knowledge followed by forming innovative approaches to the calculation of physical and mechanical properties applied to East and West Siberian fields.

Logging Methods

The mud logging tools (Toropetsky, K.V. et al., 2017) often have a more detailed depth resolution than those of standard logging and cover a wider depth range, including the most complex wellbore areas, such as the conductor and surface casing intervals where logging data is either absent or very limited (e.g., GR only). Also, the gathered data correlate directly with the mechanical effect on the rocks with little, if any, effect of drilling muds and the time factor. A serious concern is the methods' sensitivity to drilling schedules, bit types, challenges in core-to-mud log correlations in the coring intervals which are drilled using special core bits, unlike roller cone or PDC bits used for drilling the remaining intervals. Also, mud logging is very sensitive to errors often associated with poor quality of jobs (which depend on professional competence of operators and interpreters and the equipment used).

Logging While Drilling (LWD) (Toropetsky, K.V., Kayurov, N.K. et al., 2017) is in many ways similar to the standard logging suite, except that measurements are taken in the reservoirs least affected by drilling muds. Modern log suites allow measurements of sufficiently high quality (comparable to wireline or tubing/coiled tubing-conveyed logging tools). The negative aspects include the lack of acoustic measurements while drilling (almost nonexistent in Russia), as well as problems related to the operation of clamping tools (for example, gamma-ray density log).

Open-hole logging (Toropetsky, K.V., Ulyanov, V.N. et al., 2017) is the main downhole measurements that provide the distribution of geo-mechanical parameters along a wellbore. The study completeness should be one of the main requirements. According to the previous studies, the main tools used include acoustic (preferably broadband cross-dipole), Gamma-Ray Density, Thermal Neutron-Neutron logs (THPN), Gamma-Ray logs, Resistivity logs (induction and direct current methods), Caliper, less often - Spontaneous Potential. It is also important to select the tools with regard to the features in the section in question (SP, IL/VIKIZ (High Frequency Isoparametric Induction Sounding)), THPN in clastic reservoirs; Laterolog, GR-N, etc.-in carbonate reservoirs).

Also, special open-hole tools, such as downhole microimagers, pulsed neutron-neutron and neutron-gamma-ray logs, can be of great value. Besides, stimulation tools, such as formation testers, special well tests (FIT, LOT), hydraulic fracturing, etc. can provide useful information as well.

Seismic methods, such as land shooting or downhole VSP, usually provide information on the velocity of elastic waves propagation in the medium, and in order to obtain density, porosity, and other physical and mechanical properties, special Vp/Vs-based correlations should be used. Note poor vertical level of detail (>30 m) and a low contrast of the properties obtained through inversion of onshore seismic data.

Cluster Analysis in Geophysics

Clustering is one of the approaches to building petrophysical correlations (Aggarwal, C.C. et al., 2014). Essentially, clustering is a way to classify data according to a combination of petrophysical, geophysical, petrographic, geological, or other features. The most intuitive clustering example is rock typing. Since the totality of points belonging to a single cluster exhibits some physical and/or geological generality, there is reason to believe that the petrophysical bonds will be more hard expressed within a cluster than in the total dataset. Note that when identifying lithologies, interpreters primarily take into account the relative changes in geophysical parameters, i.e. changes in logging curve signatures.

A number of published sources describe the tasks of geological classifications that have been developed to predict the physical parameters of rocks using cluster analysis algorithms (Abbas Majdi et al., 2017; Yiwen Gong et al., 2019; Omid Saeidi et al., 2014), however, in our work, we will use clustering for geomechanical modeling.

In the general case, this is a research problem, however, with all the advantages of the clustering approach, incorrect identification of a point (i.e. when a correlation for a cluster is applied to a point that is erroneously (or accidentally) assigned to another cluster) may cause problems. To reduce the negative effect of incorrect cluster identification, the so-called fuzzy clustering algorithms should be applied which essentially fit the weight of each cluster membership in each point; in controversial cases (unclear membership, i.e. of several clusters with close weights simultaneously), we can choose the most reasonable and consistent version of the correlation.

Cluster Analysis Methods and Algorithms

Preliminary clustering is primarily intended to distinguish groups of points in a general dataset by a certain characteristic: within a group, points should be more similar to each other in this characteristic than points of various groups. The following must be defined for clustering:

1. The metric specifying the distance between points in multidimensional space:
   $D_{ij} = \left(\sum \left(X_i^k - X_j^k\right)^{\alpha}\right)^{1/\alpha}$, the Mahalanobis distance,
2. The method of calculating cluster centers: arithmetic mean/median or medoid (central point of the dataset).

   All clustering parameters are subject to preliminary normalization:

   $\tilde{X} = \left(X - \overline{X}\right)/\sqrt{D}$, where $\overline{X}$ is the mean, D is the variance.

Rigid clustering uniquely associates each object in the dataset with a certain cluster, in contrast to soft (or fuzzy) clustering, when each object is associated with the degree of each cluster membership (Vorontsov, K.V., 2007).

Clustering Algorithms can be descending, which divide the dataset into components according to a certain rule, and ascending (agglomeration), which, on the contrary, combine points into clusters according to a certain rule (Vorontsov, K.V., 2007).

The clustering principles are as follows

1. Topological connectance (shortest nonclosed path)
2. Centroids (K-MEANS and its variations, FOREL)
3. Balancing the distribution function (EM-algorithm)
4. Balancing the point density function (DBSCAN)
5. Spectral and correlation analysis
6. Hierarchical methods - agglomeration and separation.

A simplified classification of cluster analysis methods is shown in Figure 1.

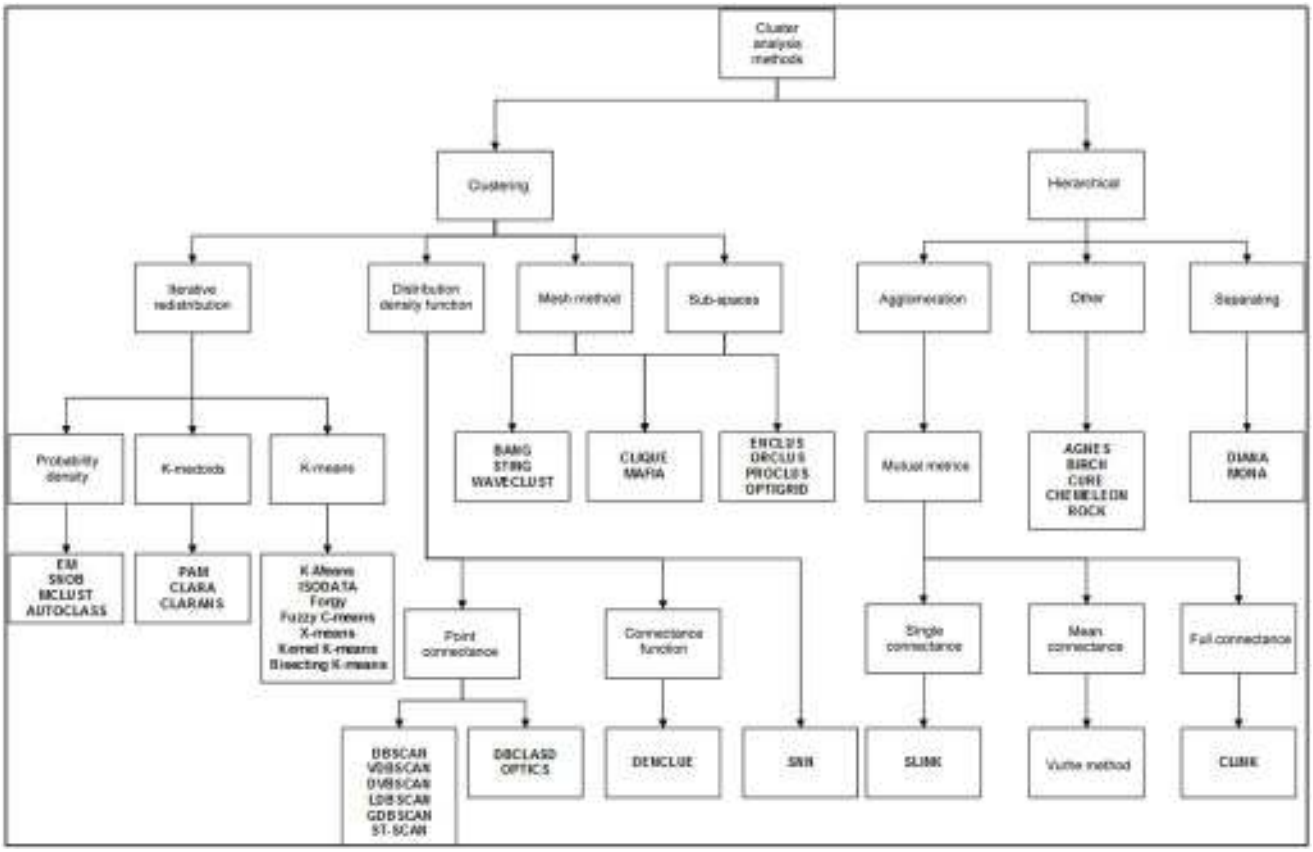


Figure 1—Classification of cluster analysis methods.

A simultaneous use of several independent clustering algorithms increases the reliability and reduces the likelihood of clustering errors, i.e. if two independent algorithms yield close clustering, we can conclude with confidence that there are valid patterns behind this clustering, and the rest of the points lie in the area of unclear membership and, in fact, carry informational "noise" which is the reason for poor-quality petrophysical correlations.

Some algorithms impose certain constraints on the shape of clusters: the centroids-based method builds clusters in the form of hyperspheres, while agglomeration algorithms are free of such constraints.

K-MEANS algorithm and its variations and the EM algorithm are very sensitive to the choice of initial cluster centers and may work inconsistently, but they have a high convergence rate compared to hierarchical ones. Hierarchical algorithms, on the contrary, require more computations of $O(N^3)$ for agglomeration and $O(2^{N-1})$ for clustering methods, but they work much more steadily in comparison with K-MEANS and the EM algorithms with $O(N^2)$ level of complexity.

A visual example of clustering is lithotype definition, i.e. rocks with a certain range and ratio of physical characteristics. In turn, clustering is a further evolution of rock typing approaches and it allows taking into account a certain range of properties, including mechanical ones, etc., in other words, to identify mechanical types of rocks (mechanotypes).

To visualize multidimensional data and clustering results in multidimensional space, the common Multidimensional Scaling (MDS) technique is used, i.e. when a set of multidimensional points is matched, according to a certain rule, with a set of two-dimensional points keeping the order of pairwise distances (Vorontsov, K.V., 2007; Zinoviev, A. Yu. 2000). Usually, a stress functional is built that expresses the total difference in pairwise distances which is then minimized. Thus, multidimensional scaling displays an arbitrary multidimensional variety in the form of a two-dimensional one while preserving topological and metric relationships.

Below is a simplest linear MDS example. Let D be the matrix of squared pairwise distances ($n \times n$)

1. Centering transformation: $B = -\frac{1}{2} J \cdot B \cdot J$; where $J = I - \frac{1}{n} E$, I is an identity matrix ($n \times n$) and E is the matrix of units ($n \times n$)
2. Next, we find the eigenvalues of the B matrix in descending order and select the first m values: λ_1, …, λ_m and the corresponding eigenvectors e_1, …, e_m
3. Finally, we find the projection of the original set onto the-dimensional plane of the principal components: $X = E_m \cdot \Lambda_m^{1/2}$, where E_m is the matrix of eigenvectors, Λ_m is the diagonal matrix of the B matrix eigenvalues.

For clustering, the distribution function of parameters or pairwise distances between points must be first analyzed to identify multiple modality. For this, multi-pick analysis tools are used. The distribution function is usually lognormal:

$$fX(x, x_0, \sigma) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\frac{(lnx - lnx_0)^2}{2\sigma^2}\right)$$

To identify mechanical lithotypes (mechanotypes), such geophysical parameters as DTP/DTS/RHOB (or equivalent dynamic elastic constants of YMD/PRD), as well as shale volume (VSH), porosity or hydrogen saturation (TNPH or log (TNPH)) must be considered in the first place.

To ensure the best clustering of a dataset, the clustering features must be selected very carefully. The basis for such selection is formed by analyzing the distribution function of each of the potential features for multimodality. On the contrary, the inclusion of a poorly contrasting feature can lead to the loss of clustering algorithms' stability, or even to inadequate clustering. For clustering, features with minimal cross-correlation should be selected; to do this, a matrix of pairwise correlations should be preliminarily built.

To evaluate the clustering quality (i.e., completeness of dataset clustering), the following functionals can be introduced:

$$\Phi_0 = \frac{\sum_{i<j} \rho(x_i x_j)\left[y_i \neq y_j\right]}{\sum_{i<j} 1\left[y_i \neq y_j\right]} - \text{inter-cluster distance functional}$$

$$\Phi_1 = \frac{\sum_{i<j} \rho(x_i x_j)\left[y_i = y_j\right]}{\sum_{i<j} 1\left[y_i = y_j\right]} - \text{intra-cluster distance functional}$$

Thus, clustering at the functional level is a problem of joint optimization (dataset clustering): $\Phi_0 \rightarrow min$ at $\Phi_1 \rightarrow max$.

Figure 2 shows an example of the Φ_0 functional versus the number of clusters for the K-MEANS, FOREL, and LANCE-WILLIAMS algorithms for the testing dataset.

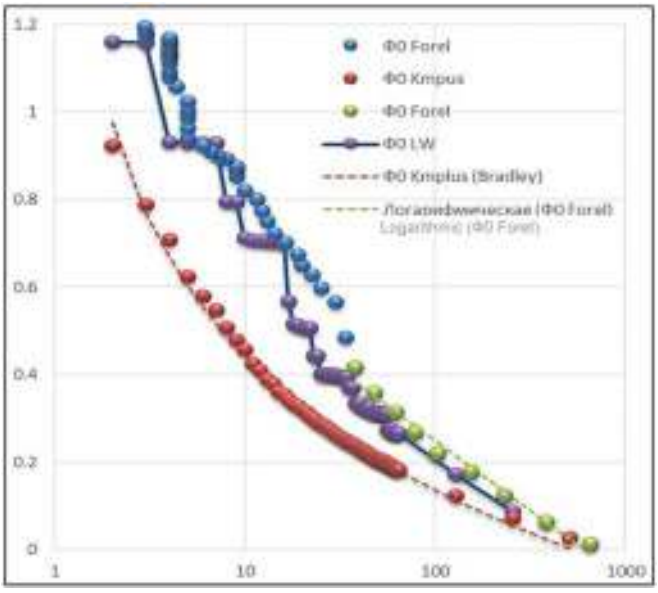


Figure 1—Φ_0 functional vs. the number of clusters for the K-MEANS, FOREL, and LANCE-WILLIAMS algorithms.

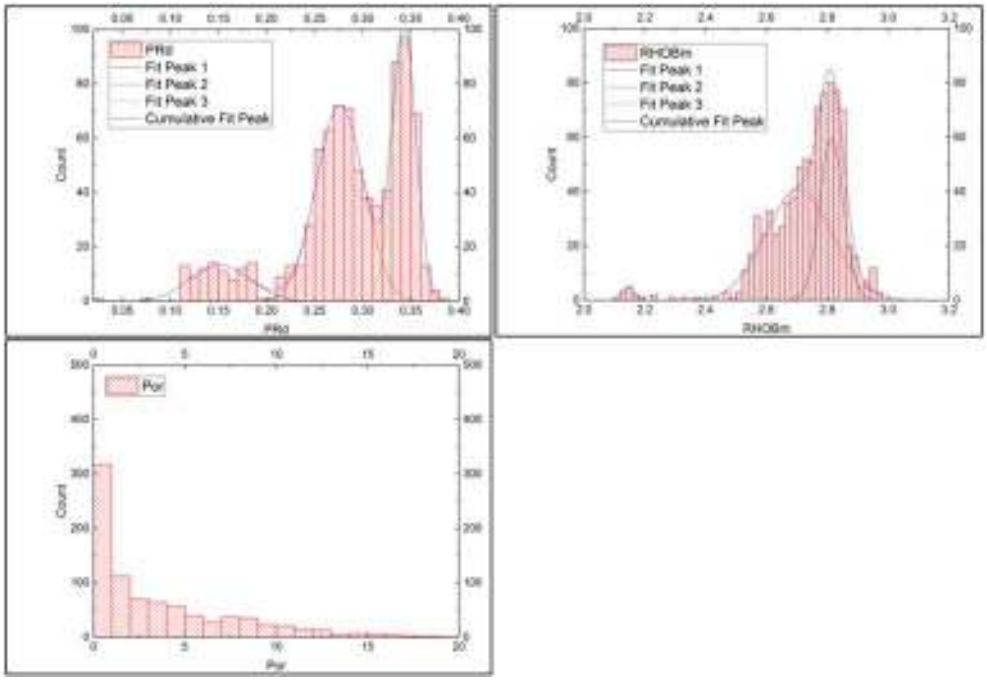


Figure 2—Distributions of petrophysical properties (PRd, RHOBm and Por) among core samples (PRd - dynamic Poisson's ratio, RHOBm - mineralogical density, Por - porosity).

One can see that for the same number of clusters, K-MEANS algorithm shows the smallest value of Φ_0, FOREL shows the largest value, and LANCE-WILLIAMS is in the middle, and with an increase in the number of clusters for all the algorithms, the Φ_0 functional monotonously reduces to zero (with the number of clusters equal to the number of points in the dataset when each point is in an isolated cluster).

Note that the $\Phi_0(N)$ functions demonstrate stepwise jumps for FOREL and LANCE-WILLIAMS, which we believe correspond to the detail of the cluster structure in the initial dataset. However, for high $N>30$, for all algorithms, $\Phi_0(N)$ behaves without any specific features, which means that increasing the number of clusters does not increase the level of detail the cluster structure, but simply splits the clusters into smaller ones.

For K-MEANS: over the entire range $\Phi_0(N) \sim \Phi_0 - k \cdot \ln(\ln(N))$ or $\Phi_0(N) \sim \Phi_0 \cdot (N + \varepsilon)^{-k}$ For FOREL and LANCE-WILLIAMS, asymptotically $\Phi_0(N) \sim \Phi_0 - k \cdot \ln(N)$

Figure 3 shows the distribution of petrophysical properties (PRd, RHOBm, and Por) among core samples. The PRd distribution shows at least three peaks, RHOBm has two overlapping peaks and one clearly isolated, three in total; Por does not contain any significant features. Thus, it is possible to assume at least 3x3 = 9 sub-sets (clusters).

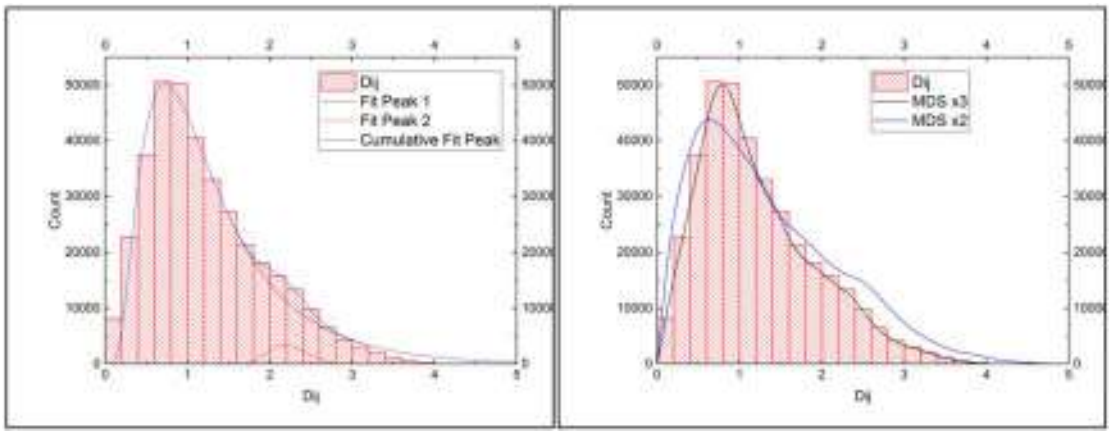


Figure 3—A histogram of pairwise distances between dataset points. A superposition of a bimodal lognormal distribution (left). MDS in the original dimension and reduced down to 2 (right).

A histogram of pairwise distances between dataset points with a superimposed bimodal lognormal distribution is shown in Figure 4 (left). The result of MDS in the original dimension and a reduction down to 2 are shown in Figure 4 (right).

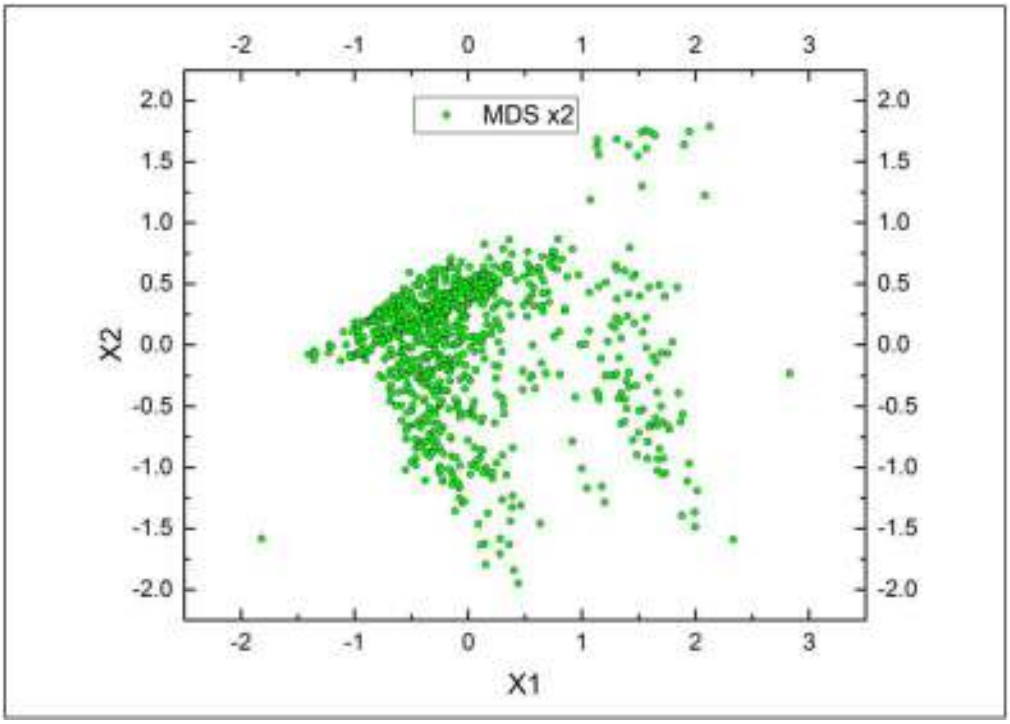


Figure 4—Applying MDS to reduce the dimension down to 2 to display the cluster structure on a cross-plot. X1 and X2 are the first two main components.

Figure 5 shows the result of applying MDS to reduce the dimension down to 2 in order to display the cluster structure on a cross-plot where X1 and X2 are the first two main vector components.

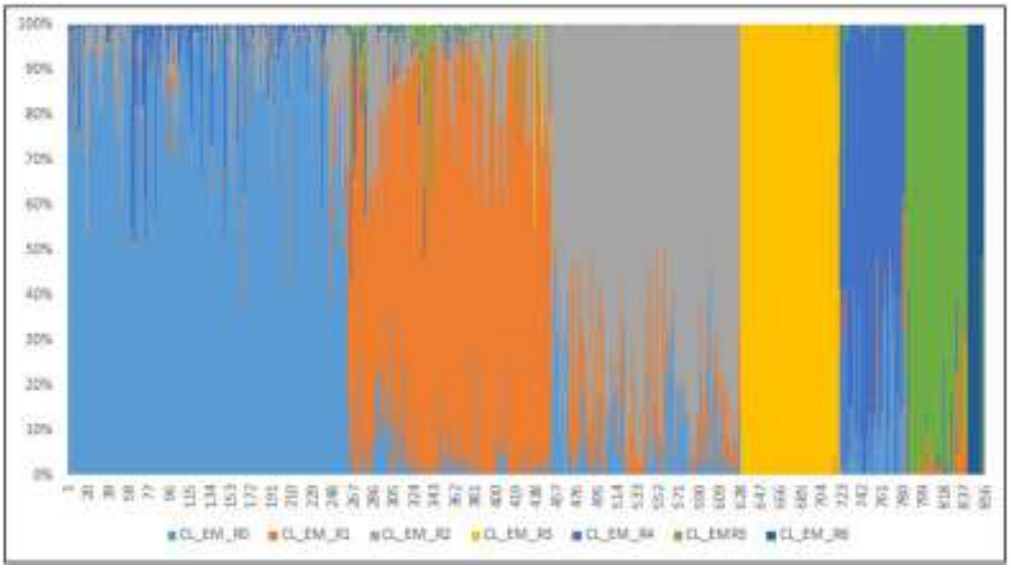


Figure 5—An example of fuzzy clustering using the EM algorithm. PRd, Por, and RHOBm selected as basic features.

Figure 6 shows an example of fuzzy clustering using the EM algorithm where PRd, Por, and RHOBm are selected as the basic features.

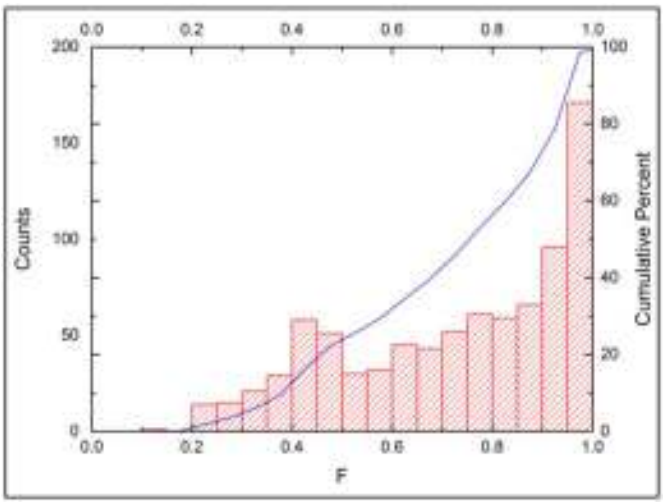


Figure 6—Histogram of the F function with superimposed cumulative distribution.

Fuzzy clustering by the EM algorithm assigns a weight vector to each point for each of the clusters. It is convenient to rank the weight vector in descending order of weight, thus sequentially listing the clusters of the first, second, etc. significance level. The diagram shows that there are almost unambiguous (clear) clustering zones and zones of unclear membership with a high weight of clusters of the second and higher significance level. It is convenient to introduce the so-called clustering clarity parameter showing how accentuated are the weights in the weight vector, for example:

$$F = \frac{1}{n-1}\sum_{i>j}\left(w_i - w_j\right)^2$$

F = 0 for fully fuzzy clustering (membership of all clusters with equal weights) and F = 1 for fully clear clustering (membership of a single cluster with weight = 1).

Figure 7 shows a histogram of the F function with superimposed cumulative distribution which demonstrates that no more than 30% of the points are equally likely to belong to two neighboring clusters, no more than 10% - to three clusters; and no more than 3% - to four clusters, respectively.

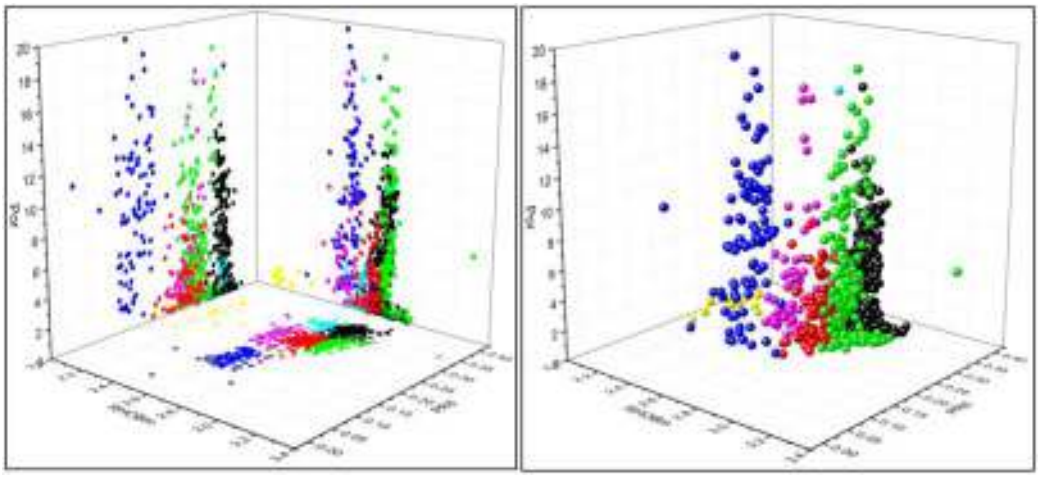


Figure 7—An example of hard clustering using the EM algorithm.

An analysis of the F distribution function contains the most important information about the cluster-based structure of the dataset. There are many ways to switch from fuzzy to hard clustering based on weight vectors, the simplest one being the majority principle, i.e. a point explicitly refers to the cluster with the maximum weight (in fact, the cluster of the first significance level).

Application of Clustering Methods

Below is an example of hard clustering using the EM algorithm, where PRd, Por, and RHOBm are taken as the basic qualities. Figure 8 shows the three-dimensional image (right) and two-dimensional projections onto the base planes (left), dataset points are colored in accordance with the cluster membership.

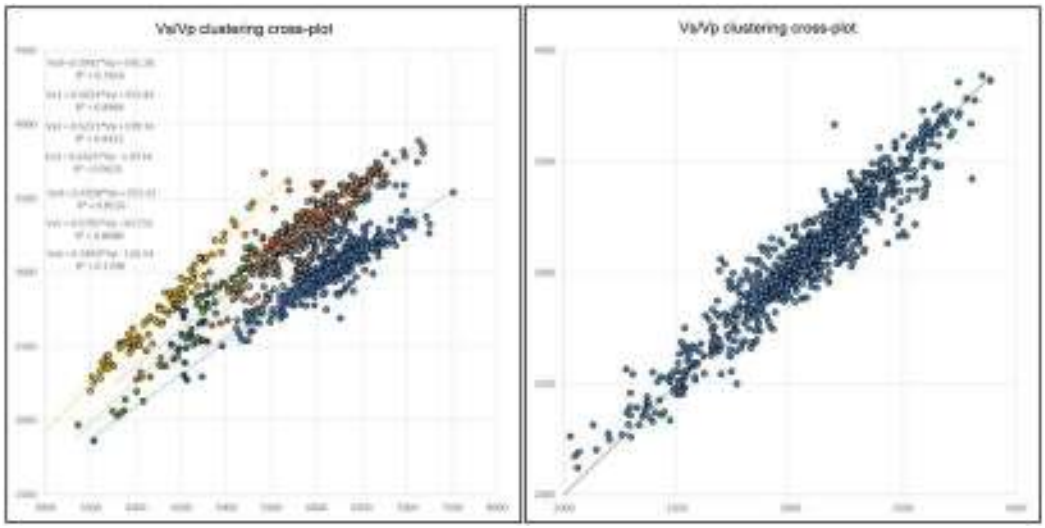


Figure 8—Vs-Vp clustering cross-plot according to selected features with superimposed trends of petrophysical correlations (left). The general cross-plot of the initial Vs and Vs restored from the set of correlations (right).

The color code: black - tight solid vuggy dolomite; red - anhydritized dolomite with veins of organic matter; green - dolomite with veins of organic matter; blue - fine-grained sandstone, siltstone; turquoise - tight solid dolomite; crimson - coarse-grained sandstone; yellow – rock salt.

Figure 9 shows the Vs-Vp cross-plot with clustering based on specified features and with superimposed trends of petrophysical correlations (left), as well as the general cross-plot of the initial Vs and the Vs restored from the set of correlations (right).

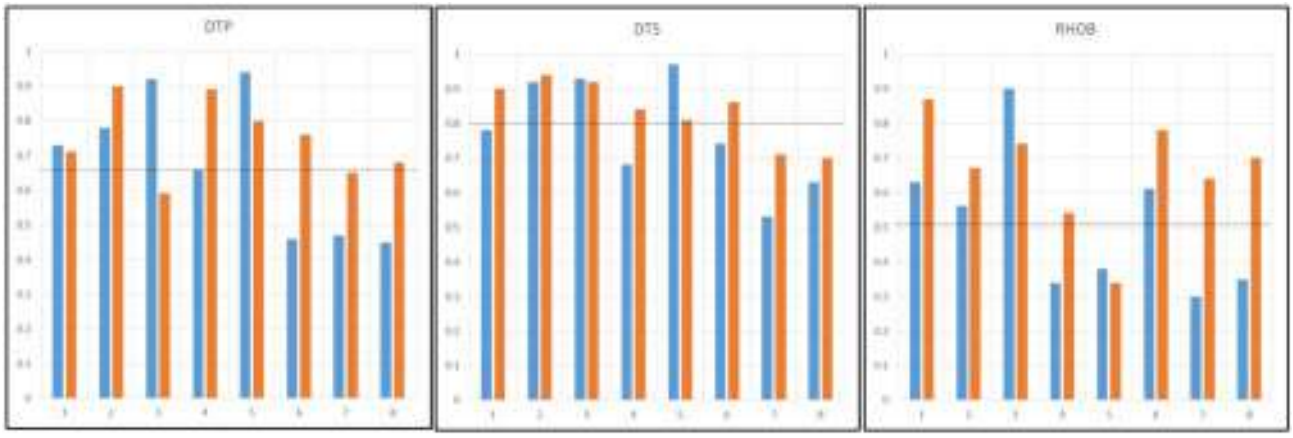


Figure 9—Comparison of the maximum achievable R^2 for the correlations on each parameter from the list (DTP, DTS and RHOB) for the selected lithotypes and corresponding cluster types; the dashed line is a non-clustered dataset.

Clustering can significantly improve the determination coefficient from 0.60 for a non-clustered dataset to 0.80 and higher for almost all lithotypes (the best results for sandstones are 0.96), except for rock salts. According to the table, a and b linear regression coefficients vary within a wide range of 0.39-0.64 and -126-591, respectively. Note that in the general dataset, the a and b parameters are not the average between the a and b coefficients for separate clusters: the a coefficient is noticeably less than the average (even less than the minimum), and the b coefficient, on the contrary, is noticeably higher than the average (even higher than the maximum). The regression model parameters for the full dataset and for separate clusters are shown in Table 1.

Table 1—Regression model parameters for full dataset and for separate clusters

Cluster	Lithotype	Parameter *a*	Parameter *b*	R^2
R0	Tight solid vuggy dolomite	0.39	591	**0.78**
R1	Anhydritized dolomite with veins of organic matter	0.50	354	**0.90**
R2	Dolomite with veins of organic matter	0.52	139	**0.84**
R3	Fine-grained sandstone, siltstone	0.64	-2	**0.96**
R4	Tight solid dolomite	0.43	353	**0.85**
R5	Coarse-grained sandstone	0.58	-51	**0.87**
R6	Rock salt	0.59	-126	**0.17**
All	All	0.32	1251	**0.60**

Table 2 (see below) gives an example of regression analysis for each parameter (DTP, DTS, RHOB), including a factor diagram (multiple linear regressions), cross plots for the training and testing datasets, the maximum achievable R^2 (for optimum correlation). A similar regression analysis is made for the non-

clustered dataset in general, and for each interpreted lithotype or cluster type, and finally for the clustered dataset.

Table 2—Example regression analysis for each parameter (DTP, DTS, and RHOB)

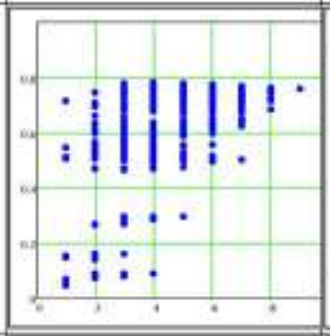
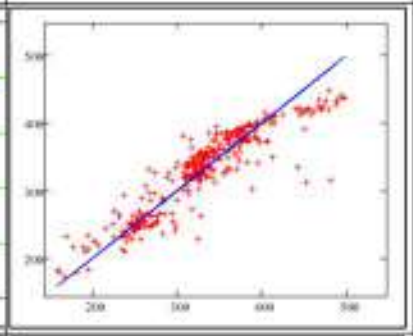
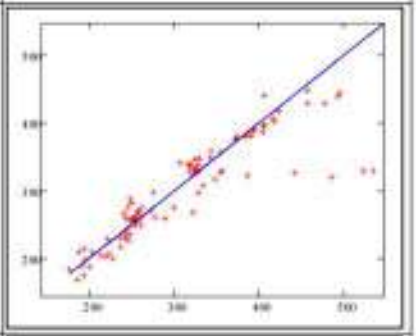
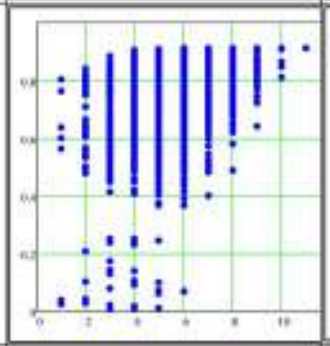
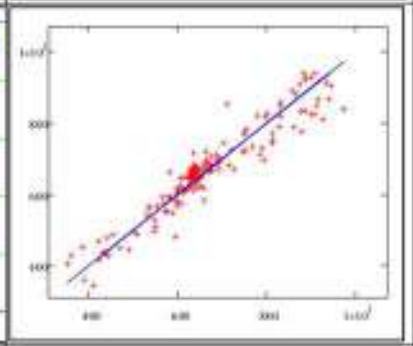
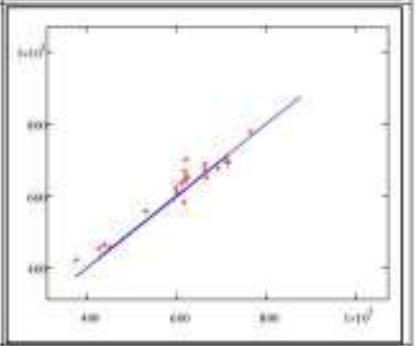
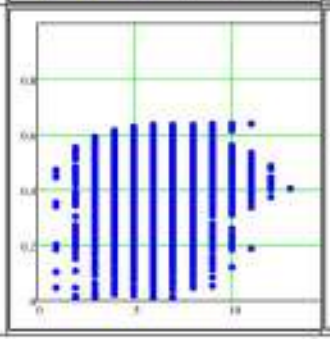
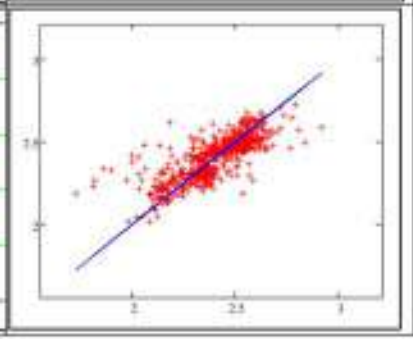
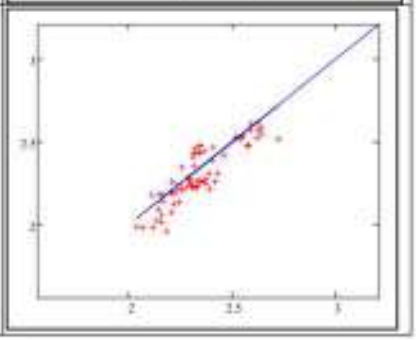
Parameter	R^2	Component Diagram	Learning Dataset Cross-plot	Testing Dataset Cross-plot
DTP	0.78			
DTS	0.90			
RHOB	0.63			

Figure 10 compares the maximum achievable R^2 for the correlations on each parameter from the list (DTP, DTS, and RHOB) for the interpreted lithotypes and the corresponding cluster types. Note that the maximum achievable R^2 for cluster types is nearly always higher than in the corresponding lithotypes.

Conclusions

In our study, we made a comprehensive analysis of world practices on the methods for calculating geomechanical properties using well logging, mud logging, and seismic data. We proposed internally-developed innovative approaches to assessing the significance of input logging parameters and selecting the most reliable correlations for calculating mechanical properties; we also analyzed dataset clustering according to various criteria to improve the correlations. We used a case study of the West Siberian clastic interval to demonstrate that petrophysical correlations improve significantly when interpreting pure

sandstones and siltstones, whereas such correlations exhibit low reliability for coals and carbonate-cemented sandstones.

The main challenge associated with the cluster analysis application is the lack of algorithms that allow repeated clustering of new datasets (selections) in the same way as it has been done on the learning dataset, with keeping the continuity of the cluster types and thereby ensuring clustering reproducibility. In all cases, learning and testing datasets were jointly clustered; therefore, the matching of cluster types in both datasets is guaranteed. We believe that a solution to this problem would lie in the development of conditional clustering algorithms, i.e. joint clustering of two datasets which preserves the clustering of the original (reference) dataset.

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